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Mystery of Ramanujan Number: $a^3 + b^3 = c^3 + d^3$

Author: Debajit Das

ABSTRACT

1729 is the least number which can be expressed as sum of two cubes in two different ways. It was first identified by the great mathematician Sri Srinivasa Ramanujan in 1913. In my solution of Beal Equation mystery which was published in August Edition 2013 in IJSER, 4th volume, it was clearly shown that there cannot exist any number which can be expressed as sum of two numbers both are in power form of equal even exponents greater than two in two or more ways. So among all odd exponents three is the minimum. Why 1729 is the least? What should be the characteristic of a number to produce such type of relation? This paper basically contains the characteristic of a number with respect to its factors so that it can produce such type of Ramanujan relation.

Keywords

Ideal Ramanujan number, Ramanujan factor, Ramanujan number, Ramanujan relation, Wings.

1. Introduction

Suppose, $N = a^3 + b^3 = c^3 + d^3$. Obviously, $N.p^3 = (ap)^3 + (bp)^3 = (cp)^3 + (dp)^3$. But it has no significance unless Np^3 produces another wing/expression of $e^3 + f^3$ i.e. $N.p^3 = (ap)^3 + (bp)^3 = (cp)^3 + (dp)^3 = e^3 + f^3$. So, there exists two kinds of numbers, may be said as Ideal Ramanujan Number(IRN) where all expressions are free from any common factor and other can be said as simply Ramanujan Number(RN) where the expressions have common factor in between the elements and at least one expression is free from any common factor.

An IRN cannot have a prime factor in power form. It is the product of m nos. of prime numbers & symbolically represented by IRN_m and a RN is always produced by p^3.(IRN) where p may be prime or composite.

By unit digit analysis of the factors involved in IRN we can identify some cases or numbers where it fails to produce any R-multi relations.

Functional form of all IRN can also be established in three categories.

2. If a positive composite odd number N = xyz having at least three prime factors is such that $R_f = \{12$(product of any two factors, say xy) $- 3z^2\}$ gives the result of square integer in 'n' nos. of cases considering all the sets of (x, y, z) then N can be expressed as sum of two cubes in 'n' nos. of ways i.e. $N = a^3 + b^3 = c^3 + d^3 = e^3 + f^3 =$ ………… n nos. of such expressions.

$N = xyz = \alpha^3 + \beta^3 = (\alpha + \beta)(\alpha^2 + \beta^2 - \alpha\beta)$ (Say)

$\Rightarrow y = \alpha + \beta$ & $\alpha^2 + \beta^2 - \alpha\beta = xz$

$\alpha^2 + (y - \alpha)^2 - \alpha(y - \alpha) = xz$ or, $3\alpha^2 - 3y\alpha + y^2 - xz = 0$

$\Rightarrow \alpha, \beta = 1/6[\,3y \pm \sqrt{\{9y^2 - 12(y^2 - xz)\}}\,]$

Obviously, $12xz - 3y^2$ is to be an odd square integer or by cyclic order we can say $12xy - 3z^2$ or $12yz - 3x^2$ will be an odd square integer.

Considering all the sets of (x, y, z) and all the three Ramanujam's factors (R_f) in each set if we get a square integer in 'n' nos. of cases then, obviously, N will create a relation like $N = a^3 + b^3 = c^3 + d^3 = e^3 + f^3 =$ ……….. n nos. of such expressions, may be said as Ramanujan's multi relation for n > 1.

This square integer, obviously, is in the form of $9(2k - 1)^2$, where k = 1, 2, 3, ……..

So, in a composite number of several factors we can divide all the factors into two groups i.e. $N = (P_i)(Q_j)$ so that $R_f = 12(P_i) - 3Q_j^2$ & P_i must have at least two factors. If R_f is square integer then α, β will be both positive or of opposite signs will depend upon $y^2 > xz$ i.e. $Q_j^2 > P_i$ or $Q_j^2 < P_i$

All though $1 = 1^n$ but by convention 1 can also be considered as one factor of IRN.

$\Rightarrow$ for the factors 1, x, y (x, y are prime nos. & x < y) there exists only two valid R_f i.e. $R_f = 12(xy) - 3.1^2$ & $R_f = 12(y) - 3.x^2$. Obviously first one will produce two roots of opposite signs whereas second one will produce two roots of both positive signs.

Hence, IRN_2 cannot produce more than two wings. It will produce only two wings where one wing is of positive elements & other wing will contain elements of opposite sign say α & – β where obviously, α & β are consecutive two numbers, otherwise IRN_2 cannot be the product of two prime nos.

Now, for three prime factors $R_f > 0$ or, $4N/z > z^2$ or, $N > z^3/4$ i.e. $N > (\text{any factor})^3/4$ required for the selection of z. Or, more strictly, $R_f \geq 9$

$\Rightarrow z^3 + 3z - 4N \leq 0$ or, $N \geq (z^3 + 3z)/4$

IRN_3 can produce a multi relation of 3 wings but one wing must be of negative expression.

3. A composite number in the form of $N = 5\{u_1(P_1)^p.u_1(P_2)^q\ldots\ldots\}\{u_9(P_1)^r.u_9(P_2)^s\ldots\ldots\}$ where $u_k(P)$ denotes a prime number of unit digits k, can never produce Ramanujan's multi relations.

As u_3 & u_7 cannot produce a square number, so from $R_f = 12P - 3Q^2$ we can say whatever may be the two groups of P & Q among the factors of N, R_f will always produce a number of nature u_3 or u_7.

4. A composite number which is a product of three prime numbers $N = u_p(P_1).u_q(P_2).u_r(P_3)$ can produce an ideal R-multi relations of 2 wings only when p, q, r all are different excluding 5.

N can produce an ideal R-multi relations of 3 wings in the following repeated cases

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$\{u_1(P), u_1(P), u_7(P)\}$, $\{u_1(P), u_1(P), u_9(P)\}$, $\{u_3(P), u_3(P), u_1(P)\}$, $\{u_3(P), u_3(P), u_7(P)\}$, $\{u_7(P), u_7(P), u_3(P)\}$, $\{u_7(P), u_7(P), u_9(P)\}$, $\{u_9(P), u_9(P), u_1(P)\}$, $\{u_9(P), u_9(P), u_3(P)\}$ or $p = q = r \neq 5$

We can establish it by unit digit analysis for the acceptance or rejection of a group of R_f as shown below.

4.1 When all the unit digits are different.

u_p, u_q, u_r	u_k of R_f	Accept/Reject	Remarks
1, 3, 5	$12(u_1)(u_3) - 3(u_5)^2 = u_6 - u_5 = u_1$	Accepted	Fails to produce multi-relation
	$12(u_1)(u_5) - 3(u_3)^2 = u_0 - u_7 = u_3$	Rejected	
	$12(u_3)(u_5) - 3(u_7)^2 = u_0 - u_3 = u_7$	Rejected	
3, 5, 7	By similar approach	All are rejected	Fails to produce multi-relation
5, 7, 9	$12(u_5)(u_7) - 3(u_9)^2 = u_0 - u_3 = u_7$	Rejected	Fails to produce multi-relation
	$12(u_7)(u_9) - 3(u_5)^2 = u_6 - u_5 = u_1$	Accepted	
	$12(u_9)(u_5) - 3(u_7)^2 = u_0 - u_7 = u_3$	Rejected	
7, 9, 1	$12(u_7)(u_9) - 3(u_1)^2 = u_6 - u_3 = u_3$	Rejected	Can produce a multi-relation.
	$12(u_9)(u_1) - 3(u_7)^2 = u_8 - u_7 = u_1$	Accepted	
	$12(u_7)(u_1) - 3(u_9)^2 = u_4 - u_3 = u_1$	Accepted	
7, 9, 3	$12(u_7)(u_9) - 3(u_3)^2 = u_6 - u_7 = u_9$	Accepted	Can produce a multi-relation.
	$12(u_9)(u_3) - 3(u_7)^2 = u_4 - u_7 = u_7$	Rejected	
	$12(u_7)(u_3) - 3(u_9)^2 = u_2 - u_3 = u_9$	Accepted	
1, 5, 7	$12(u_1)(u_5) - 3(u_7)^2 = u_0 - u_7 = u_3$	Rejected	Fails to produce multi-relation
	$12(u_5)(u_7) - 3(u_1)^2 = u_0 - u_3 = u_7$	Rejected	
	$12(u_1)(u_7) - 3(u_5)^2 = u_4 - u_5 = u_9$	Accepted	
1, 9, 5	By similar approach	All are rejected	Fails to produce multi-relation
1, 3, 9	$12(u_1)(u_3) - 3(u_9)^2 = u_6 - u_3 = u_3$	Rejected	Can produce a multi-relation.
	$12(u_3)(u_9) - 3(u_1)^2 = u_4 - u_3 = u_1$	Accepted	
	$12(u_1)(u_9) - 3(u_3)^2 = u_8 - u_7 = u_1$	Accepted	
1, 3, 7	$12(u_1)(u_3) - 3(u_7)^2 = u_6 - u_7 = u_9$	Accepted	Can produce a multi-relation.
	$12(u_3)(u_7) - 3(u_1)^2 = u_2 - u_3 = u_9$	Accepted	
	$12(u_1)(u_7) - 3(u_3)^2 = u_4 - u_7 = u_7$	Rejected	
3, 5, 9	$12(u_3)(u_5) - 3(u_9)^2 = u_0 - u_3 = u_7$	Rejected	Fails to produce multi-relation
	$12(u_5)(u_9) - 3(u_3)^2 = u_0 - u_7 = u_3$	Rejected	
	$12(u_3)(u_9) - 3(u_5)^2 = u_4 - u_5 = u_9$	Accepted	

4.2 When two unit digits are equal and one is different.

u_p, u_q, u_r	u_k of R_f	Accept/Reject	Remarks
1, 1, 3	$12u_1(x)u_1(y) - 3\{u_3(z)\}^2 = u_5$	Accepted	Fails to produce multi-relation
	$12u_1(x)u_3(z) - 3\{u_1(y)\}^2 = u_3$	Rejected	
	$12u_1(y)u_3(z) - 3\{u_1(x)\}^2 = u_3$	Rejected	
1, 1, 5	By similar approach	All rejected	Fails to produce multi-relation
1, 1, 7	By similar approach	All accepted	Can produce a multi-relation.
1, 1, 9	By similar approach	All accepted	Can produce a multi-relation.
3, 3, 1	By similar approach	All accepted	Can produce a multi-relation.
3, 3, 5	By similar approach	All rejected	Fails to produce multi-relation
3, 3, 7	By similar approach	All accepted	Can produce a multi-relation.
3, 3, 9	$12u_3(x)u_3(y) - 3\{u_9(z)\}^2 = u_5$	Accepted	Fails to produce multi-relation
	$12u_3(x)u_9(z) - 3\{u_3(y)\}^2 = u_7$	Rejected	
	$12u_3(y)u_9(z) - 3\{u_3(x)\}^2 = u_7$	Rejected	
5, 5, 1	$12u_5(x)u_5(y) - 3\{u_1(z)\}^2 = u_7$	Rejected	Can produce a multi-relation.
	$12u_5(x)u_1(z) - 3\{u_5(y)\}^2 = u_5$	Accepted	
	$12u_5(y)u_1(z) - 3\{u_5(x)\}^2 = u_5$	Accepted	
5, 5, 3	$12u_5(x)u_5(y) - 3\{u_3(z)\}^2 = u_3$	Rejected	Can produce a multi-relation.
	$12u_5(x)u_3(z) - 3\{u_5(y)\}^2 = u_5$	Accepted	
	$12u_5(y)u_3(z) - 3\{u_5(x)\}^2 = u_5$	Accepted	
5, 5, 7	$12u_5(x)u_5(y) - 3\{u_7(z)\}^2 = u_3$	Rejected	Can produce a multi-relation.
	$12u_5(x)u_7(z) - 3\{u_5(y)\}^2 = u_5$	Accepted	
	$12u_5(y)u_7(z) - 3\{u_5(x)\}^2 = u_5$	Accepted	
5, 5, 9	$12u_5(x)u_5(y) - 3\{u_9(z)\}^2 = u_7$	Rejected	Can produce a multi-relation.
	$12u_5(x)u_9(z) - 3\{u_5(y)\}^2 = u_5$	Accepted	
	$12u_5(y)u_9(z) - 3\{u_5(x)\}^2 = u_5$	Accepted	
7, 7, 1	$12u_7(x)u_7(y) - 3\{u_1(z)\}^2 = u_5$	Accepted	Fails to produce multi-relation
	$12u_7(x)u_1(z) - 3\{u_5(y)\}^2 = u_7$	Rejected	
	$12u_7(y)u_1(z) - 3\{u_7(x)\}^2 = u_7$	Rejected	
7, 7, 3	By similar approach	All accepted	Can produce a multi-relation.
7, 7, 5	By similar approach	All rejected	Fails to produce multi-relation

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7, 7, 9	By similar approach	All accepted	Can produce a multi-relation.
9, 9, 1	By similar approach	All accepted	Can produce a multi-relation.
9, 9, 3	By similar approach	All accepted	Can produce a multi-relation.
9, 9, 7	$12u_9(x)u_9(y) - 3\{u_7(z)\}^2 = u_5$	Accepted	Fails to produce multi-relation
	$12u_9(x)u_7(z) - 3\{u_9(y)\}^2 = u_3$	Rejected	
	$12u_9(y)u_7(z) - 3\{u_9(x)\}^2 = u_3$	Rejected	
9, 9, 5	By similar approach	All rejected	Fails to produce multi-relation

4.3 When all the three factors have the same unit digits.

By similar approach here all are accepted.

In all the cases as tabulated above, if there exists any common factor among (u_p, u_q, u_r**)**, it is to be ignored.
N = 5.5.u_k (K ≠ 5) fails to produce any relation as because R_f is in the form of 5.u_k (K ≠ 5) which cannot be a square integer.
For IRN_k, k > 3, each group of three factors will follow the same rule of unit digits.

4. Minimum number that produces R-multi relations

4.1 Minimum number for IRN_2.

Among all the prime numbers, we get first IRN_2 as a product of two prime factors i.e. N = 7.13 where $13 > (7/2)^2$ and R_f is found to be square integer in both the cases.
i.e. $R_f = 12(7.13) - 3.1^2 = 33^2$ & $R_f = 12(13.1) - 3.7^2 = 3^2$.
Accordingly, α, β = 1/6[3.1 ± 33] = 6, – 5 & γ, δ = 1/6[3.7 ± 3] = 4, 3.
Hence, $7.13 = 91 = 6^3 + (-5)^3 = 4^3 + 3^3$ which is the least.
91 can be symbolically written as IRN_2(1+, 1–)

4.2 Minimum number for IRN_3.

We know $N \geq (z^3 + 3z)/4$ where z is the greatest factor among three. Considering equality symbol put z = 3, 5, 7, 9, ….. We first receive N as a product of three acceptable prime factors for z = 19 i.e. N = 7.13.19 where there is a possibility of getting R-multi relations.
Here, $R_f = 12.(7.13) - 3.19^2 = 3^2$,
$= 12.(7.19) - 3.13^2 = 33^2$,
$= 12.(13.19) - 3.7^2$ = obviously not a square integer = $(53.07)^2$
As we have received square integer in two cases 1729 is capable of producing R-multi relations of two wings.
Now from α, β = 1/6[3y ± $\sqrt{\{12xz - 3y^2\}}$] we get α, β = 1/6[3.19 ± 3] = 10, 9
& α, β = 1/6[3.13 ± 33] = 12, 1
⇒ $1729 = 1^3 + 12^3 = 9^3 + 10^3$ which is the least and known as Ramanujan number.
1729 can be symbolically written as IRN_3(2+, 0–)

5. If a number produces a relation $N = a^3 + b^3 = c^3 + d^3$ then $N.p^3$ must produce a relation $N = (ap)^3 + (bp)^3 = (cp)^3 + (dp)^3$ and by virtue of this multiplier p3 an additional wing may be produced or may not. If not produced, there is virtually no change of N.

As $N = a^3 + b^3 = c^3 + d^3$, R_f will produce square integer in two cases.
Say $R_f = 12(xy) - 3z^2 = I_1^2$ & $R_f = 12(xz) - 3y^2 = I_2^2$
Now, $R_f.p^3 = 12(xp.yp) - 3(zp)^2 = (I_1p)^2$
Again $R_f.p^3 = 12(zp.xp) - 3(yp)^2 = (I_2p)^2$
⇒ a, b = 1/6[zp ± I_1p] & are changed to ap, bp
Similarly, c, d = 1/6[yp ± I_2p] & are changed to cp, dp
⇒ $N = (ap)^3 + (bp)^3 = (cp)^3 + (dp)^3$
Now by virtue of this multiplier p^3 if other combination where there does not lie any common factor among the three factors e.g. $12(xyp) - 3(zp^2)^2$ produces an additional wing $e^2 + f^2$ then the new born number has some significant meaning i.e. $Np^3 = (ap)^3 + (bp)^3 = (cp)^3 + (dp)^3 = e^3 + f^3$. Otherwise it has no meaning.
Depending upon the different nature of multi relations we can classify all the Ramanujan numbers in the following two ways.

IRN_f (m+, n-): Ideal Ramnujan number produces m nos. of expressions with positive elements & n nos. of expressions where one element is negative and all the expressions are free from any common factor in between the elements & f denotes the number prime factors present in the number.
RN_f (m+, n-): Ramanujan number can be defined by same but some pair of expressions will show a common factor in between the elements and there exists at least one expression without any common factor & f denotes the number prime factors present in the number excluding p^3.

Example:
Say, N = 7.31.67.223.
Here total nos. of three groups are six.
7.31, 67, 223 i.e. u_7, u_7, u_3 combination & can produce R-relation.
7.67, 31, 223 i.e. u_9, u_1, u_3 combination & can produce R-relation.
7.223, 31, 67 i.e. u_1, u_1, u_7 combination & can produce R-relation.
31.67, 7, 223 i.e. u_7, u_7, u_3 combination & can produce R-relation.
31.223, 7, 67 i.e. u_3, u_7, u_7 combination & can produce R-relation.
67.223, 7, 31 i.e. u_1, u_1, u_7 combination & can produce R-relation.
Out of these six cases we get R_f as a square integer in the following three cases.

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$12(7.67.223) - 3(31)^2 = 1119^2 = (3.373)^2$,
$12(67.223) - 3(7.31)^2 = 195^2 = (3.65)^2$ &
$12(7.31.67) - 3(223)^2 = (3.53)^2$
$\Rightarrow$ a, b = $1/6[3.31 \pm 1119] = 202, -171$
c, d = $1/6[3.7.31 - 195] = 76, 141$
e,f = $1/6[3.223 \pm 159] = 138, 85$
Hence, $(7.31.67.223) = 3242197 = 202^3 + (-171)^3 = 76^3 + 141^3 = 138^3 + 85^3$
And symbolically this number can be denoted by $IRN_4(2+, 1-)$

Now consider a number $87539319 = 3^3.7.31.67.223$
$12(9.7.67.223) - 3(3.31)^2 = 3357^2 \Rightarrow \alpha, \beta = 1/6[3.3.31 \pm 3357] = 606, -513$
$\Rightarrow 87539319 = 606^3 + (-513)^3 = 3^3\{202^3 + (-171)^3\}$
Again, $12(9.67.223) - 3(21.31)^2 = 585^2 \Rightarrow \alpha, \beta = 1/6[3.21.31 \pm 585] = 423, 228$
$\Rightarrow 87539319 = 423^3 + 228^3 = 3^3(141^3 + 76^3)$
Again, $12(9.7.31.67) - 3(3.223)^2 = 477^2 \Rightarrow \alpha, \beta = 1/6[3.(3.223) \pm 477] = 414, 255$
$\Rightarrow 87539319 = 414^3 + 255^3 = 3^3(138^3 + 85^3)$

Now significant $R_f = 12(3.7.31.223) - 3(9.67)^2 = 807^2 \Rightarrow \alpha, \beta = 1/6[3.9.67 \pm 807] = 436, 167$
$\Rightarrow 87539319 = 436^3 + 167^3$
Hence, $N = 87539319 = 3^3.7.31.67.223 = (606)^3 + (-513)^3 = (423)^3 + (228)^3 = 414^3 + 255^3 = 436^3 + 167^3$
i.e. $N = (3.202)^3 + (-3.171)^3 = (3.141)^3 + (3.76)^3 = (3.138)^3 + (3.85)^3 = 436^3 + 167^3$
And symbolically this number can be denoted by $RN_4(3+, 1-)$

6. An ideal Ramanujam number cannot have a prime factor in power form and any Ramnujan number is the product of an IRN or RN with p^3, p being an odd integer.

We can say, P_1^{n1} cannot produce IRN or RN, P being a prime number.
$\Rightarrow P_1^{n1} P_2^{n2}$ also fail to produce any IRN or RN.
$\Rightarrow P_1^{n1} P_2^{n2} P_3^{n3}$ also fail to produce any IRN or RN & so on.
So in general $N = P_1^{n1} P_2^{n2} P_3^{n3}$…… fail to produce any IRN or RN.
For IRN, n1 = n2 = n3 = ……. = 0 and there exists at least three prime numbers.
For RN there must be a multiplier p^3 (p is odd) with any IRN or RN produced.

If N is an ideal Ramanujan number, then whether $N.p^3$ will produce a R-number or not, it depends upon the distribution of p, p, p among the three factors of a group.

Group of three factors where p is equally distributed (xp, yp, zp) will always produce a Rmanujan relation like $(ap)^3 + (bp)^3 = (cp)^3 + (dp)^3$ This relation will get a significant meaning by producing $(ap)^3 + (bp)^3 = (cp)^3 + (dp)^3 = e^3 + f^3$ (there is no c.f in between e & f) only when at least one R_f is found to be a square integer by unequal distribution of p among the three factors.

7. Minimum of INR_2 with respect to its leading & lowest factor.

Say, N = xy where x < y and $R_f = 12(xy) - 3.1^2$ & $R_f = 12(y) - 3.x^2$.
Accepted cases are:

S.No.	$N = x.y = u_i(P).u_j(P)$	Least Number
1.	$u_7(P).u_1(p)$	$N = 7.31 = 217 = 6^3 + 1^3 = 9^3 + (-8)^3$
2.	$U_1(P).u_7(p)$	$N = 31.277 = 8587 = 19^3 + 12^3 = 54^3 + (-53)^3$
3.	$u_7(P).u_3(p)$	$N = 7.13 = 91 = 3^3 + 4^3 = 6^3 + (-5)^3$
4.	$U_1(P).u_9(p)$	Not found within f(40, 1) and g(40, 1)
5.	$U_9(P).u_1(p)$	Not found within f(40, 1) and g(40, 1)
6.	$U_3(P).u_9(p)$	$N = 13.79 = 1027 = 10^3 + 3^3 = 19^3 + (-18)^3$
7.	$U_1(P).u_1(p)$	Not found within f(40, 1) and g(40, 1)
8.	$U_3(P).u_3(p)$	$N = 43.733 = 31519 = 31^3 + 12^3 = 103^3 + (-102)^3$
9.	$U_9(P).u_9(p)$	Not found within f(40, 1) and g(40, 1)

8. Functional form of IRN.

In view of all facts & figures as stated above we can divide the functional form of all IRN into following three categories.
$f(x, k) = \{(2x) - (2x - k)\}\{(2x)^2 + (2x - k)^2 + 2x(2x - k)\} = k(12x^2 - 6kx + k^2)$
where k is an odd prime number including one or product of multi prime nos. & < 2x.
$g(x, k) = \{(2x + k) - (2x)\}\{(2x)^2 + (2x + k)^2 + 2x(2x + k)\} = k(12x^2 + 6kx + k^2)$
where k is an odd prime no. including one or product of multi prime nos.
$h(x, k) = \{(k - 2x) + (2x)\}\{(k - 2x)^2 + (2x)^2 - 2x(k - 2x)\} = k(12x^2 - 6kx + k^2)$
where k is an odd prime number or product of multi prime nos. > 2x.
Here, f(x, k) will always produce an expression like $(\text{odd integer})^3 - (\text{even integer})^3$.
g(x, k) will always produce an expression like $(\text{even integer})^3 - (\text{odd integer})^3$ &
h(x, k) will always produce an expression like $(\text{even integer})^3 + (\text{odd integer})^3$
So, for Ramaujan multi relations all the leading expressions will start from either f(x, k) or g(x, k) or h(x, k)

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The leading expression whose prime factors are suitable for producing R_f as a square integer will generate more wings, may be positive expression or negative expression for f(x, k) & g(x, k) but for h(x, k) all are bound to be of positive expressions.
Let us extract few examples in favor of all the three functions.

8.1 f(x, k)

x =	f(x, 1)	f(x, 3)	f(x, 5)	f(x, 7)	f(x, 11)	f(x, 13)	f(x, 15)	f(x, 17)	f(x, 19)
1	7								
2	37	63							
3	91 *	189	215						
4	169	387	485	511					
5	271	657	875	973					
6	397	999	1385	1603	1727				
7	547	1413	2015	2401	2717	2743			
8	721 *	1899	2765	3367*	3971	4069	4095		
9	919	2457	3635	4501	5489	5707	5805	5831	
10	1141	3087	4625	5803	7271	7657	7875	7973	7999
11	1387	3789	5735	7273	9317	9919	10305	10523	10621
12	1657	4563	6965	8911	11627	12493	13095	13481	13699
13	1951	5409	8315	10717	14201	15379	16245	16847	17233
14	2269	6327	9785	12691	17039	18577	19755	20621	21223
15	2611	7317	11375	14833	20141	22087	23625	24803	25669
16	2977	8379	13085	17143	23507	25909	27855	29393	30571
17	3367*	9513	14915	19621	27137	30043	32445	34391	35929
18	3781	10719	16865	22267	31031	34489	37395	39797	41743
19	4219	11997	18935	25081	35189	39247	42705	45611	48013
20	4681	13347	21125	28063*	39611	44317	48375	51833	54739

X* are capable of producing additional wings.
$f(3, 1) = 91 = 7.13 = 3^3 + 4^3 = 6^3 + (-5)^3$
$f(8, 1) = 721 = 7.103 = 16^3 + (-15)^3 = 9^3 + (-2)^3$
$f(17, 1) = 3367 = 7.13.37 = 16^3 + (-9)^3 = 34^3 + (-33)^3 = 15^3 + (-2)^3$
$f(20, 7) = 28063 = 7.19.211 = 31^3 + (-12)^3 = 40^3 + (-33)^3$

8.2 g(x, k)

x =	g(x, 1)	g(x, 3)	g(x, 5)	g(x, 7)	g(x, 11)	g(x, 13)	g(x, 15)	g(x, 17)	g(x, 19)
1	19	117	335	721	2189	3367*	4905	6851	9253
2	61	279	665	1267	3311	4849	6795	9197	12103
3	127	513	1115	1981	4697	6643	9045	11951	15409
4	217*	819	1685	2863	6347	8749	11655	15113	19171
5	331	1197	2375	3913	8261	11167	14625	18683	23389
6	469	1647	3185	5131	10439	13897	17955	22661	28063
7	631	2169	4115	6517	12881	16939	21645	27047	33193
8	817	2763	5165	8071	15587	20293	25695	31841	38779
9	1027*	3429	6335	9793	18557	23959	30105	37043	44821
10	1261	4167	7625	11683	21791	27937	34875	42653	51319
11	1519	4977	9035	13741	25289	32227	40005	48671	58273
12	1801	5859	10565	15967	29051	36829	45495	55097	65683
13	2107	6813	12215	18361	33077	41743	51345	61931	73549
14	2437	7839	13985	20923	37367	46969	57555	69173	81871
15	2791	8937	15875	23653	41921	52507	64125	76823	90649
16	3169	10107	17885	26551	46739	58357	71055	84881	99883
17	3571	11349	20015	29617	51821	64519	78345	93347	109573
18	3997	12663	22265	32851	57167	70993	85995	102221	119719
19	4447	14049	24635	36253	62777	77779	94005	111503	130321
20	4921*	15507	27125	39823	68651	84877	102375	121193	141379

X* are capable of producing additional wings.
$g(4, 1) = 217 = 7.31 = 6^3 + 1^3 = 9^3 + (-8)^3$
$g(9, 1) = 1027 = 13.79 = 10^3 + 3^3 = 19^3 + (-18)^3$
$g(20, 1) = 4921 = 7.19.37 = 17^3 + 2^3 = 41^3 + (-40)^3$

Note: leading expression is to be considered for least value of k. e.g. 3367 has appeared in three cases
f(17, 1), f(8, 7) & g(1, 13) where k is least for f(17, 1). So, $34^3 + (-33)^3$ is the leading set. In each case algebraic sum of the elements equals to k.

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8.3 h(x, k)

x =	h(x, 3)	h(x, 5)	h(x, 7)	h(x, 11)	h(x, 13)	h(x, 17)	h(x, 19)	h(x, 23)	h(x, 29)
1	9	35	133	737	1339	3383	4921	9269	19691
2		65	91*	407	793	2261	3439	6923	15689
3			217*	341	559	1547	2413	5129	12383
4				539	637	1241	1843	3887	9773
5				1001	1027*	1343	1729*	3197	7859
6					1729*	1853	2071	3059	6641
7						2771	2869	3473	6119
8						4097	4123	4439	6293
9							5833	5957	7163
10								8027	8729
11								10649	10991
12									13949
13									17603
14									21953

X* are capable of producing additional wings.
h(6, 13) = 1729 = 7.13.19 = $12^3 + 1^3 = 10^3 + 9^3$
Note: any number of multi relations having at least one positive expression is made available by h(x, k).
K = 5or multiple of 5 and 3 or multiple of 3 in all the functions can be ignored.
IRN of n wings = $(P_1P_2P_3\ldots..)^m.(P'_1P'_2P'_3\ldots..)$ where P denotes a prime number, implies that for n ≥ 2, m = 0 and for n = 1, m = 0 or 3 and 2nd part is $.(P'_1P'_2P'_3\ldots..)^q$ where q = 1 or 2.

Conclusion:

Whether or not, a number is capable of producing Ideal Ramanujan multi-relation and thereafter Ramanujan multi-relation is fully dependent on the factors does the number have. If the factors are conducive to the existence of R_f as a square integer at least in two cases, the number will produce R-multi relations. But is there any relation among all the factors. It needs further investigation.

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Books

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Author: Debajit Das
(Company: Indian Oil Corporation Ltd, Country: INDIA)